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THIS JOURNAL IS DEDICATED TO THE FOLLOWING AIMS: (1) Through published standard papers on the culture aspects, humanism and history of mathematics to deepen and to widen public interest in its values. (2) To supply an additional medium for the publication of expository Mathematical articles. (3) To promote more scientific methods of teaching mathematics. (4) To publish and to distribute to groups most interested high-class papers of research quality representing all mathematical fields.

Every paper on technical mathematics offered for publication should be submitted (with enough enclosed postage to cover two two-way transmissions) to the Chairman of the appropriate Committee, or to a Committee member whom the Chairman may designate to examine it, after being requested to do so by the writer. If approved for publication, the Committee will forward it to the Editor and Manager at Baton Rouge, who will notify the writer of its acceptance for publication. If the paper is not approved the Committee will so notify the Editor and Manager, who will inform the writer accordingly.

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Mathematics and Nature

Such symbols as 1, 2/3, 0, $\sqrt[n]{7}$, $\sqrt{-1}$, $a+bi$, $a+bi+cj$, taken in this order, may reasonably be regarded as indexing the most significant germinal phases in the historic development of number concepts. Ordered thus, they mirror in simple fashion the facts (1) that the growth of mathematics implies increasing generalization of its concepts, (2) the widening of the fields covered by its generalizations seems to render the concepts increasingly void of meaning in the domain of everyday reality. Millions of the objective-minded—represented by the housekeeper, the merchant, or the bank clerk—know the indispensableness of 1, 2/3, and even 0, or similar arithmetic symbols. Never-theless, what banker or merchant or farmer has a use for $\sqrt{-1}$, or $7-10\sqrt{-1}$?

But—and here is the *heart* of the present note—no process of the human mind can conceivably be more *errorless in its logic* than the mathematical generalization which has been examined and approved by a myriad of mathematicians and disapproved by none.

(1) Alongside of this perfection of logical consistency in the mathematical generalization consider the following two truths:

(2) In a final analysis, the material of mathematics reduces to the logic of relationships, let these be of things real, things conceptual, or both.

(3) Nature is a vast domain of real relationships.

Then, noting the common factors of (1), (2), (3), should we be surprised that the flawless logic of relationships in mathematical processes should have counterparts in the logic of nature's real relationships?

Do we wonder that, while $\sqrt{-1}$ may have no value for the baker or the candlestick maker, a profound relationship of this imaginary unit to the domain of the "real" has made it valuable in electrical science?

Need it astonish us that Riemann should have had the opinion that a science of physics could not have existed "before the invention of differential equations?" Should it be surprising that the quaternion, $a+bi+cj+dk$, a purely conceptual creation of W. R. Hamilton, should have become a valuable tool in physical investigations?

Finally, what else but these common properties of the mathematical processes and the processes of *nature* could account for the fruitful results of the application of the potential function to problems in electricity, magnetism, and gravitational attraction?

S. T. Sanders.

Concerning a Certain Web of Conic

H. L. Dorwart
Washington and Jefferson College

The three types of pencils (one-parameter families) of central conics whose semi-axes are the trigonometric ratios of the Pythagorean relations are illustrated by the following equations:

$$(1)\qquad \frac{x^2}{\lambda^2-1}+\frac{y^2}{\lambda^2}=1=\frac{y^2}{\lambda^2}-\frac{x^2}{1-\lambda^2}\ ,$$

which represents confocal conics with foci $(0, \pm 1)$. Ellipses are given by $\lambda = \sec x$ or $\operatorname{cosec} x$,* and hyperbolas by $\lambda = \sin \xi$ or $\cos \xi$.

$$(2)\qquad \frac{x^2}{1-\lambda^2}+\frac{y^2}{\lambda^2}=1=\frac{y^2}{\lambda^2}-\frac{x^2}{\lambda^2-1}\ ,$$

which represents conics touching the four lines $x+y-1=0$, $x-y+1=0$, $x+y+1=0$, $x-y-1=0$. Ellipses are given by $\lambda = \sin \theta$ or $\cos \theta$ and hyperbolas by $\lambda = \sec \theta$ or $\operatorname{cosec} \theta$,

$$(3)\qquad \frac{x^2}{\lambda^2-1} - \frac{y^2}{\lambda^2}=1.$$

This family of hyperbolas for $\lambda = \sec \xi$ or $\operatorname{cosec} \xi$ is neither confocal nor does it have a real envelope. Since

$$\lim_{\lambda\to\infty} \frac{\lambda}{\sqrt{\lambda^2-1}}=1,$$

it has the property that its limiting member is an equilateral hyperbola. However, the hyperbolas of (1) and (2) also have this property. A better description is in terms of related families. If we call hyperbolas and ellipses having the same semi-axes *associated* conics, then associated with the ellipses touching four lines are the confocal hyperbolas, while associated with the confocal ellipses are the hyperbolas touching four lines and the family (3). A further interesting connection between families (1) and (2) comes from the following. The locus of the poles of the line $x+y-1=0$ with respect to the different members of the confocal family (1) is easily shown to be $x-y+1=0$,

*All angles are restricted to values between 0° and 90°.

and the locus of poles of $x-y+1=0$ is $x+y-1=0$. Likewise for $x-y-1=0$ and $x+y+1=0$. Hence these four lines are invariant (but not line-wise) under the pole and polar relationship with respect to the confocal conics

These families are of course well known, although the specialization of the parameter leads to some interesting geometrical constructions.*

A fact that might not be guessed in advance is that the differential equations of which (1), (2) and (3) are the primitives are closely related. They are

$$p^2xy+(\ \ x^2-y^2+1)p-xy=0 \tag{1'}$$

$$p^2xy+(-x^2-y^2+1)p+xy=0 \tag{2'}$$

$$p^2xy+(-x^2-y^2-1)p+xy=0, \tag{3'}$$

and each is reducible to a Clairaut equation by the substitutions

$$x^2=X,\quad y^2=Y. \tag{4}$$

Hence an examination of the conditions under which the differential equation

$$p^2xy+(Dx^2+Ey^2+F)p+Gxy=0 \tag{5}$$

reduces to a Clairaut equation for these substitutions, and a discussion of the resulting curves may be of some interest.

Applying (4) to (5), one readily obtains

$$XP^2+DXP+EYP+FP+GY=0,$$

or, solving for Y,

$$Y=\frac{-P-D}{EP+G}PX-\frac{FP}{EP+G}\ . \tag{6}$$

If $E=-1$ and $G=-D$, this reduces to the Clairaut equation

$$Y=PX+\frac{FP}{P+D} \tag{7}$$

whose singular solution (envelope) is

$$(Dx^2-y^2+F)^2+4Dx^2y^2=0, \tag{8}$$

*See *Families of Conics with Trigonometric Parameters* by H. L. Dorwart. The Mathematics Teacher.

and whose general solution is

$$y^2 = Cx^2 + \frac{FC}{C+D} \,. \tag{9}$$

Since (9) is of the second degree, it represents nothing more complicated than conics; but considering it as a web (three-parameter family), a rather careful analysis is necessary in order to determine the properties of its members. We note that $C+D$ must be different from zero, and that $F=0$ or $C=0$ causes the conics to degenerate into straight lines. For $C<0$, the conics are ellipses and for $C>0$ they are hyperbolas, the slopes of the asymptotes being $\pm\sqrt{C}$.

$D=0$ gives a net (two-parameter family) of conics through the points $(0, \pm\sqrt{F})$ for positive F, while for negative F the only real curves are hyperbolas with semi-conjugate axes equal to $\sqrt{-F}$. The most interesting case for $D>0$ occurs when $D=1$, giving a net of confocal conics with foci at $(0, \pm\sqrt{F})$ for $F>0$ and at $(\pm\sqrt{-F}, 0)$ for $F<0$. The differential equation (5) with $E=-1$ and $D=-G=1$, displays the well known fact that such a confocal family is self-orthogonal.

Finally, suppose $D<0$. For $F>0$, (8) represents a real, non-degenerate envelope. Putting $D=-r^2$ and $F=s^2$, this envelope is readily seen to be made up of the four lines $rx-y+s=0$, $rx+y-s=0$, $rx+y+s=0$, $rx-y-s=0$, and hence for this case we have the web of conics touching these four lines. For $F<0$, the only real curves are hyperbolas. For a fixed D, as C approaches $|D|$, the slopes of the asymptotes approach $\pm\sqrt{D}$. Hence these curves can be described as a net of hyperbolas, the slopes of whose asymptotes approach limiting values.

Thus (9) is seen to be an equation of more than passing interest, representing as it does, for appropriate values of the constants, most of the common families of central conics.

"Puzzles and paradoxes have been popular since antiquity, and in amusing themselves with these playthings men sharpened their wits and whetted their ingenuity. But it was not for amusement alone that Kepler, Pascal, Fermat, Leibniz, Euler, Lagrange, Hamilton, Cayley, and many others devoted so much time to puzzles. Researches in recreational mathematics sprang from the same desire to know, were guided by the same principles, and required the exercise of the same faculties as the researches leading to the most profound discoveries in mathematics and mathematical physics. Accordingly, no branch of intellectual activity is a more appropriate subject for discussion than puzzles and paradoxes."
—From ***Mathematics and the Imagination*** by Edward Kasner and James Newman. pp. 156-157.

Comments Concerning "A Note on Observed Geometric Series" by A. B. Soble in the April, 1940 Issue.

By William Dowell Baten
Michigan State College

Let the predicting equation concerning cigarette production be as follows:

$$y_i = r y_{i-1}, \quad (i=2,3,\cdots, 13).$$

Then the best value of r according to the least squares method is

$$r = \sum_{i=2}^{i=13} y_i y_{i-1} \Big/ \sum_{i=1}^{12} y_i^2$$

$$=643.8346/537.7804 = 1.1972.$$

If the predicting equation is

$$y_i/r = y_{i-1};$$

then the value of r according to the least squares method is

$$r = \sum_{i=2}^{13} y_i^2 \Big/ \sum_{i=2}^{13} y_i y_{i-1}$$

$$=772.4956/643.8346 = 1.1998.$$

If the predicting equation is

$$r = y_i/y_{i-1}$$

then the best value of r according to the least squares method is

$$r = (1/12)\sum_{i=2}^{13} (y_i/y_{i-1}) = 13.9068/12 = 1.1589.$$

Sople's value is 1.1814.

Since this series of tobacco production figures is assumed to be a geometric series, the correct value of the ratio r is found by accumulating the 1901 production, 2.72, to the 1913 production 15.56 by the compound interest law. This is

$$15.56 = 2.72\, r^{12},$$

from which $r = 1.1564.$

According to the definition of a geometric mean, the value of r is the average value for the period or the value which accumulates the first value in the series to the last value. The only one of the above values of r that will do this is the one obtained from the compound interest law. The 1913 figures obtained by accumulating 2.72 at the above rates are as follows:

$$2.72(1.1972)^{12} = 23.58$$

$$2.72(1.1998)^{12} = 24.20$$

$$2.72(1.1589)^{12} = 15.96$$

$$2.72(1.1814)^{12} = 20.11.$$

It is interesting to note that the third or the value obtained from the arithmetic average of the ratios y_t/y_{t-1} comes, for these data, closer to the real result, 15.56, than any of the other values obtained by the least squares method. The values obtained by the least squares method are not the geometric mean of this series and should not be used as such. Sople's method is interesting, however, it does not lead to a geometric mean in all cases.

Dr. Charles William Berry assistant professor of mathematics in Columbia University, is scheduled to be at Lawrence College this fall as associate professor of mathematics. He succeeds Professor John Lymer, who is retiring after thirty-seven years of service on the Lawrence College faculty. —Reported by L. J. Adams.

Four University of Chicago professors, two of them native Chicagoans, will become professors emeritus today (October 1) on reaching the age of 65.

The retiring professors, whose service on the University faculty totals 123 years are:

Dr. Fred C. Koch, Frank P. Hixon distinguished service professor and chairman of the department of biochemistry, internationally recognized authority on hormones, enzymes, and vitamins. Under Dr. Koch's direction Dr. L. C. McGee first isolated the male sex hormone.

Dr. Gilbert A. Bliss, Martin A. Ryerson distinguished service professor and chairman of the department of mathematics, leader in the field of mathematical analysis and civilian ballistics aide during the first World War.

Dr. Frederick S. Breed, associate professor of education and authority on the theory of instruction.

Dr. Franklin Bobbitt, professor of education and authority on education curriculums.

Humanism and History of Mathematics

Edited by
G. WALDO DUNNINGTON

A History of American Mathematical Journals

By BENJAMIN F. FINKEL
Drury College

(Continued from May, 1941, issue)

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small, and as the *Analyst* depends entirely upon its subscribers for the pecuniary aid necessary for its production, and for voluntary contributions to fill its pages, it was scarcely to be expected that it would survive to the age it has already attained.

It has been our intention from its birth, as we have heretofore stated to our subscribers, to continue the *Analyst* as long as our health will permit and the interest in it, manifested by its readers, continues unabated. We are, therefore, pleased to be able to say that, so far as we can judge at present, the *Analyst* will survive yet several years; and as we trust it has been of some service in promoting the cultivation of the science of mathematics, which is the key to all the other sciences, we hope the interest and support of our subscribers will continue unabated.

The Nos. of Vol. VIII shall appear promptly as they are due, No. 1, about the first of January, 1881, pp. 199-200.

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The publication of the *Analyst* was commenced with no exalted expectation of its success, as the history of like publications in this country attests the difficulty of sustaining a periodical devoted exclusively to serve and exact scientific research.

It is therefore with some degree of gratification that we are able to state that the publication has been thus far conducted without pecuniary loss.

Though our subscription list is not large yet an examination of the published volumes will show that we number among our contributors many of the ablest mathematicians and astronomers in America, besides several eminent Europeans. And it is with considerable satisfaction that we have been able to present to our readers eight annual volumes made wholly of original contributions, with a few translations. For this we take no praise to ourselves but freely acknowlage our indebtedness to our learned and liberal subscribers.

We shall commence the ninth volume of the *Analyst*, therefore, with confidence that the friends who have thus far stood by us will continue their partonage and support, and that new names will be added to our list of subscribers and contributors to Vol. IX, pp. 199-200.

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Errors of Interpolated Values Derived from Numerical Tables by Means of First Differences, by R. S. Woodward, C. E., (continued from page 149), pp. 169-175; Note on Professor Nicholson's Singular Value of π, by Professor W. W. Johnson, p. 176; Note on Experimental Confirmation of Theoretical Deduction, by the Editor, pp. 176-177; Integration of Some General Classes of Trigonometric Functions, by Professor P. H. Philbrick, Iowa State University, Iowa City, pp. 177-180; On a New Curve for the Trisection of an Angle, by Dr. William Hillhouse, pp. 181-185; New Notation for Anharmonic Ratios, by Professor William Woolsey Johnson, pp. 185-189; Solutions of Problems in No. 5, Vol. IX, pp. 189-195; Five Problems proposed for Solution, p. 195; Announcement of Vol. X, in which the Editor again renews his promise to continue the publication of the *Analyst* as long as his health permits and the interest of his subscribers is manifested by appropriate contributions to its pages, p. 196.

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Announcement. We regret to have to inform our readers that we have concluded to discontinue the publication of the *Analyst* on the completion of Vol. X. This determination has not been induced by any lack of interest in the publication manifested by our subscribers or contributors, most of whom have generously stood by us and assisted us during the whole of the ten years life of our publication, but wholly on account of our declining health. In taking leave of our contributors and subscribers, we do not propose to waste words in any attempt to apologize for the many defects in our production, but will only say that we are fully sensible of, and regret their existence, but did the best we could under the circumstances, to avoid them.

We trust that, notwithstanding its defects, the *Analyst* will be found to contain many papers of much interest and permanent value, which have been contributed by some of America's ablest mathematicians and astronomers.

No. 6 of Vol. X, the concluding number, will be issued about the first of November, and will be devoted mainly to a general index of the ten volumes published. In this index, besides correcting the errors and supplying the unintentional omissions in the published indexes, the names of contributors of questions and solutions will also be inserted.

Any subscriber, for any Volume of the *Analyst*, except Vols. I and II, who may have failed to receive, or may have subsequently lost any No. of such Vol., and who may desire to have a complete file, can have the missing Nos. supplied without charge if he will notify us in time to mail such missing Nos. with No. 6.

We have a few (about 20) complete sets of the *Analyst* which we will send to any address, free of postage, if ordered before January 1,

1884, for $15 per set; and any volume, except I and II, will be sent singly for $1.50, p. 160.

J. E. HENDRICKS.

Contents of Vol. X, No. 6.

Description of a New Ellipsograph, by the Editor, pp. 161-162; Solution of Problem 442 (see p. 155), by W. P. Casey, San Francisco, Cal., pp. 163-164; Note on Problem 443, by Professor Seitz, p. 164.

Since the above was put in type we have received from Professor J. M. Greenwood, Kansas City, Mo., the following letter announcing the death of Professor Seitz, which we take the liberty to publish, as a brief tribute to his virtue and ability, by one who knew him personally.

Kansas City, Mo., Oct. 11, 1883.

Dear Sir: The brilliant mathematician, Enoch B(eery) Seitz, died at Kirksville, Mo., on the 8th inst., of typhoid fever, after a protracted illness of five weeks. He went to the "Normal" on the day the session opened, but was unable to take charge of his classes the next day. His death causes unusual regret among the thousands of students, teachers, and citizens of this state, who admired him not only on account of his transcendent powers as a mathematician, but as a model of excellence in his daily life.

He was about 34, (b. Aug., 24, 1846), I think, though I speak from memory only.—The rising star set ere it reached the meridian.

J. M. GREENWOOD.

We have never had the pleasure of meeting Mr. Seitz and our earliest knowledge of him dates with commencement of the *Analyst*, since which the pages of the *Analyst* bear witness to his constant and valuable correspondence, from which we have long regarded him as possessed of extraordinary mathematical ability and precision of thought. And though we have been favored with the correspondence of many able mathematicians, we believe that, in acuteness of perception, and in conciseness and elegance of style, Mr. Seitz would rank with the ablest. We had anticipated valuable results from his labors, and believe that, had his life been spared, his industry and ability would have materially assisted in enlarging the boundaries of exact science.

The readers of the *Analyst* are indebted to Mr. Seitz not only for the many elegant solutions by him that have been published, but also for the "Index to Contributors of Solutions of Problems," which was furnished by him, voluntarily and unsolicited, and must have been

about the last work that he was permitted to do, as it bears date Sept. 5.

As announced in the Sept. *Analyst*, this No. will terminate the publication, under its present management. We had hoped to be able, in this issue, to answer the many inquiries that have been made as to its probable continuance, by a definite announcement of a publication to take its place, under favorable auspices and an able management, as several gentlemen of acknowledged ability, and well and favorably known by all mathematicians and astronomers, both in this country and abroad in Europe, have expressed a willingness to assume the labor and responsibility of continuing the publication; but as the arrangements for its continuance appear to be still incomplete, we have not been authorized to make a definite announcement. We feel confident, however, from the correspondence we have had on the subject, that the work will not be abandoned but will be placed on a permanent basis, under a management that will insure its usefulness and success. And we earnestly solicit for the new publication, the active assitance and patronage of all our readers.

In conclusion, we desire to tender our sincerest thanks to our patrons and contributors for their continued support during the ten years life of the *Analyst*, and especially for their kind words and manifestations of interest in our personal welfare.—EDITOR. Page 166.

It is a singular coincidence that just as the Editor of the *Analyst* was closing its stimulating and useful career, that one of its most valuable contributors should be called to lay down his powerful pen for time and eternity.

While the writer of this history was only eighteen years old when Professor Seitz died and had never heard of him at that time, yet a few years afterwards, he learned of his service to mathematics through the matchless and elegant solutions of difficult problems in the various branches of mathematics that were contributed to the mathematical journals by Professor Seitz. In studying Professor Seitz's contributions, the writer learned to love him as though he was the writer's teacher in the flesh and thus Professor Seitz, though dead, had a wonderful influence on the writer's early mathematical life. F.

Dr. Willard H. Garrett, for thirty-nine years head of the departments of mathematics and astronomy at Baker University, has been awarded the doctor of science degree by his alma mater, Illinois College. Much of his work has been in the field of testing the efficiency of the mathematics teaching in the high schools. Last September (1940) over 4000 mathematics students from twenty-three colleges in Kansas took his tests. —Reported by L. J. Adams.

The Resistance of Ships

By H. Bateman
California Institute of Technology

1. *The ship's orientation and its measurement.* A person on board ship likes to use axes of reference fixed relative to the ship and meeting at the ship's center of gravity, *G*. When the ship is in port these axes may be taken to be 1°—a horizontal axis running from stern to stem, 2°—a horizontal transverse axis pointing from the port side of the ship to the starboard or steering side, 3°—a vertical axis along the line of action of the resultant gravitational force.

As the ship moves away from port these axes change their orientations and the passengers fervently hope that the third axis will remain nearly vertical. Small departures of this axis from the vertical are specified by angles of heel and trim. When the deck is not horizontal the ship is said to have a list. This may be measured by an inclinometer which is often a plomb line or pendulum suspended against a batten but J. G. Tawresey[1] and T. U. Taylor[2] have recommended a type of reflecting pendulum which may consist of a liquid surface. The use of a U-tube as an inclinometer was advocated by A. Taylor at a meeting of the English Institute of Naval Architects in 1884 while thirty years later a similar recommendation was made by C. Comment to the corresponding German Institute. An improved form of this type of instrument was described the following year by the Japanese engineers Suyehiro and Yamamoto. An inclinometer used on airplanes contains mercury in a tube which is almost circular. The Reid aeroplane control indicator, which includes a circular U-type of inclinometer with a constriction at the bottom, is described in *Engineering*, volume 114 (1922) pp. 216-218. The position of *G* and the trim for a given ship actually depend upon the location of the ship's passengers and cargo. Indeed, one condition for equilibrium is that the ship should orient itself so that the resultant weight and the resultant force of buoyancy act along the same vertical line. A graphical method of making trim calculations and a trim nomogram are given in a paper by E. V. Telfer,[3] who suggests that Naval Architecture offers many useful applications of the principles of nomography or of

[1] J. G. Tawresey, J. So. Naval Architects and Marine Engineers, Advance paper for the meeting of Nov. 15-16 (1928) 17 pp.

[2] T. U. Taylor, The Engineer, 169: 334-335 (1940).

[3] E. V. Telfer, Engineering, 115: 565-567 (1923) discussion on p. 423.

the alignment chart. He points out that thirty years ago an early application of these principles was made by Maurice d'Ocagne who has done so much to make them known. At the instigation of N. Bertin, d'Ocagne[4] expressed in nomographic form the complicated relationship involved in a mathematical theory of the starting and stopping of a ship. The problem of starting has been discussed more recently by J. K. Whittemore[5] on the basis of various expressions for the acceleration a in terms of velocity v. The word is mathematically interesting. Numerical results are obtained in the case when

$$a = c/v - kv^n$$

and the indicated horse power H is given by $KH = D^{2/3}v^3$ where D is the displacement. The lost distance L covered in attaining full speed is then expressed in the form $L = 0.15144KD^{1/3}$, n being equal to 2. Values of K and D are taken from C. W. Dyson's *Practical Marine Engineering*, seventh edition.

The mathematical problem of the stopping of a ship is complicated by the fact that there are oscillations in the resistance when the ship passes from deep to shallow water.

2. *The ship's speed.* In a calm sea the ship will generally be driven with its first axis in the direction of motion, and power must be used to maintain the ship at a constant speed. This may be quickly estimated from measurements of the rate of revolution of a shaft and a relation known from the design and trial runs. A good idea of the relation for different types of ship may be obtained from Admiral D. W. Taylor's book *The Speed and Power of Ships*. The old direct method of measuring speed by means of a "common log" consisting of a log-ship, a log-reel, a log-line and a log-glass, has been largely superseded. Among the other direct methods which have been used we may mention first the continuously recording turbine log operated by water which enters a tube at the bottom of the ship. The Forbes instrument is briefly described in the *Enclyclopedia Britannica*. A turbine log and autograph were installed on S. S. *France* many years ago and a method was devised by which the course followed by the ship could be quickly found graphically. An H. S. V. A. log developed in Germany is described in the Annual Report for 1927 of the Hamburg Experiment Tank. It is claimed that by its means the actual instantaneous speed can be found at any place on the ship. The use of the Pitot tube for measuring a ship's speed was advocated and tried

[4] M. d'Ocagne, Bulletin Association Tech. Maritime (1911).

[5] J. K. Whittemore, Annals of Math. (2) 21: 291-298 (1919-20), Proc. Nat. Acad. Sci. 6: 182-185 (1919-20).

by Henri Pitot himself about 1732. A manometric speed indicator for ships was described by A. E. Fletcher at the meeting of the British Association for the Advancement of Science at Edinburgh in 1871, and a Committee was formed for the purpose of experimenting with instruments for measuring the speed of ships and currents. A manometric differential log depending on a combination of the Pitot or dynamic pressure tube with the Darcy-Bazin static pressure tube was patented by E. Raverot and P. Relly[6] in 1893. It was installed on three ships and experiments with it were sanctioned by authorities of the French Marine.

A ship's log and speed indicator based on the principle of the Dines anemometer has been described by S. Shimizu. It comprises a closed container enclosing a specially shaped buoy floating in mercury. Many other methods of measuring the speed have been tried or are under consideration.

In volume 81 of the *Transactions of the Institution of Engineers and Shipbuilders* in Scotland there is a paper on ships' speed meters read by H. Hoppe in 1938. A description is given of an instrument with two dials, one of which shows the instantaneous speed while the other gives the total distance traveled.

3. *Unsteady motion of a ship.* In a state of variable motion the ship's axes change their orientations relative to fixed axes and it is advantageous to consider angular velocities about fixed axes that coincide instantaneously with the ship's axes, these angular velocities being those which at each instant will carry the ship's axes into their new positions. The motions in question are described as the rolling, pitching, and yawing, of the ship.

An apparatus for automatically recording the rolling of a ship in a seaway was described by W. Froude[7] in 1872. It consisted of a flywheel three feet in diameter and so delicately supported that it would remain undisturbed when the ship rolled. This wheel, which was to weigh 200 pounds was intended to replace the rocking arm which was formerly kept level and pointing to the horizon by an observer on deck. A horizon-pen recorded the angle which the ship made with the horizon at any instant while a pendulum-pen recorded the angle which the ship made at each instant with the mean or effective surface of the wave, on which the ship was riding.

In the use of a pendulum of short period with the point of suspension at the center of gravity of the ship, Froude had been anticipated by a French naval architect. Froude's instrument was ex-

[6] E. Raverot and P. Relly, Comptes Rendus, 133:811-812 (1901).
[7] W. Froude, British Association Report (1872) 243-245.

pensive and had some defects due to friction at the pivots. Integrations involving a knowledge of the ship's dynamic constants were needed to derive the ship's motion and the wave form from the records.

An improved form of rolling indicator which may be placed anywhere in the ship was described by Mallock[8] in 1901. It consists of a paddle-wheel supported on fine pivots and enclosed in a short cylindrical box filled with fluid of the same density as the wheel so that the friction of the pivots is nearly eliminated.

In the design of this instrument Mallock tried to make use of the principle that in the case of a box completely filled with a perfect fluid no motion given to the box can communicate rotation to the fluid itself. This is not true, of course, for a real fluid like water however small the viscosity, yet the influence of viscosity is confined chiefly to a thin layer next to the boundary, particularly when the boundary is given a sudden motion or a periodic motion of high frequency. Some distance from the sides the fluid is practically undisturbed unless the motion is sufficient to produce large eddies. The rapid decay of the amplitude of oscillatory motion as the distance from the boundary increases was pointed out by Stokes[9] in a study of the effect of the internal friction of fluids on the motion of pendulums. The idea of a boundary layer in fluid motion is now quite familiar, largely on account of the mathematical work of L. Prandtl, Th. von Karman and others. The idea is expressed rather crudely in the term skin friction which Froude says was in use at least as early as 1876. When Professor A. F. Zahm was working on his bibliography on skin friction and boundary flow he remarked[10] that the origin of the term skin friction is hard to trace. I suspect that it was introduced by the physiologists and physicists who were interested in the flow of blood in capillaries.

Besides automatic recording devices there is now the automatic control which is used on many ships and airplanes. Reference may be made to the description of the Sperry Automatic Pilot[11] and to the report of H. L. Hazen[12] on the Theory of Servo-Mechanisms in which the mathematical analysis is explained in detail.

4. *The forces on a ship in motion.* The forces acting on a ship consist of gravitational forces and the surface tractions exerted by the air and water. In the case of a sailing ship the air forces are used

[8] A. Mallock, Engineering, 71:407-410 (1901).

[9] G. G. Stokes, Cambridge Phil. So. Trans. 9:8-106 (1850), Math. and Phys. Papers, 3:1-141.

[10] In a letter to the author dated May 12, 1931.

[11] E. A. Sperry, Jr., Aviation Eng. (1932) 16-18. Sci. American April (1933). G. Weinblum Schiffbau. 38:51-59 (1937).

[12] H. L. Hazen, Jn. Franklin Insti., 218:279-331, 543-580 (1934).

for propulsion, but in the case of a steamship or ship with oil as fuel they hinder the motion of the ship to some extent. Hunsaker[13] says it has been estimated that the air resistance of a 20-knot battleship is 3% of the water resistance while Mumford[14] gives the air drag of a 25-knot cross channel steamer as 2% of the water resistance. The percentages are much higher when there is a strong head wind. It is only recently that serious efforts have been made to reduce air resistance. Experimental research in wind tunnels has led to the design of streamlin d funnels such as those of the new United States liner S. S. *America* in which there are special devices to keep the smoke away from the sports deck.[15]

The surface tractions exerted by the water on the propellers and hull furnish the force of propulsion, the resistance to motion, and the force of buoyancy which keeps the ship from sinking. The last force exists even when the ship is not moving through the water, but this static force is supplemented by a dynamic force when the ship is in motion. The magnitude of this dynamic force depends upon the trim of the ship and may be quite large in the case of a speed boat with a planing bottom. In such a case the draught and displacement vary with the speed.

In a general unsteady motion the ship may slip sideways, move up and down as well as forwards and have angular velocities of roll, pitch, and yaw. When the departures from the steady state are small it may be advantageous to introduce resistance derivatives analogous to those used so successfully by G. H. Bryan[16] in his study of the stability of an airplane. The derivatives associated with small changes in orientation occurred in the studies of static stability that were made long ago and led to estimates of the stability of a vessel. Bouguer's theory of the metacenter was taken up by the geometers and amplified by many beautiful geometrical theorems which are given in the books and in a review of the subject by J. Bruhn.[17]

The static stability of a ship varies during a voyage on account of the consumption of fuel and changes in the cargo. An indicating instrument, called the Stabilograph, has been invented by Professor Abell[18] and put on the market by Dobbie-McInnes and Clyde Ltd. of Glasgow.[19] It consists essentially of a damped compound pendulum. It can be used to determine metacentric height and the period of oscillation in still water, but it is not designed to act as a rolling indicator.

[13] J. C. Hunsaker, Engineering, 118:722-733 (1924).
[14] E. R. Mumford, Engineering, 120:271-274 (1925).
[15] The Engineer, 165:166-167 (1940).
[16] G. H. Bryan "Stability in Aviation" MacMillan (1911).
[17] J. C. Bruhn, Engineering 61:488-490 (1896).
[18] T. R. Abell, Engineering, 127:484 (1929).
[19] The Engineer, 147:494-495 (1929).

The rolling, pitching and yawing of a ship are resisted or augmented by couples exerted by the water and must be taken into consideration in a study of the dynamic stability of a ship. These motions also lead to changes in the resistance to forward motion. The effect of pitching is most serious and is, indeed, the chief cause of reduction in average speed. This effect depends to some extent upon the ratio of the ship's length to the length of the waves, and so a ship which is of the right length for the waves in one ocean may not be right for another ocean. Efforts have been made to eliminate rolling by the provision of anti-rolling tanks or stabilizing gyroscopes. In this way inventors have been kind to the mathematician; for with these stabilizing devices the mathematical problem of a ship's motion under varying conditions is practically one of small oscillations about a state of steady motion, whereas the problem of large oscillations is quite hard.

An anti-rolling tank may be pictured as a kind of U-tube containing a highly viscous fluid such as oil for which a simplified hydrodynamical theory is possible. This type of stabilizer has been developed chiefly in Germany and is associated with the name of Frahm[20] while the gyroscopic type has been developed in several countries, the work of Sperry[21] in this country being particularly fine. Another type of anti-rolling device is the bilge keel which was used when the first lifeboats were designed in England 150 years ago. This device is still used on most battleships and was adopted on the *Queen Mary* though the rival Atlantic liners the *Europa, Bremen* and *Rex* had other types of stabilizing device. Bilge keels have been much studied in England and Holland with the aim of reducing their resistance. It has been found that in some cases it is advantageous to divide the bilge keel up into separate parts. Some experiemental work on bilge keels was done in this country about 1898 and is described by Spear.[22] Proof was given of the effect in increasing stability and it is interesting to note that bilge keels were added to U. S. S. *Oregon*. Bilge keels were omitted in the design of some English battleships but were eventually restored as the ships rolled badly. Some notable mathematical work on the effect of bilge keels was done by G. H. Bryan[23] who made use of the theory of discontinuous fluid motion.

[20] See for instance, O. Föppl, Ingenieur Archiv 5:35-42 (1934), H. J. R. Biles, Trans. Inst. Naval Arch. (1925), R. Malmstrom, Acta Soc. Sc. Fennicæ, 35, No. 2 (1907) 17 pp., N. Minorsky, Engineering, 158:154 (1934), H. Schreck, Marine Eng. and Shipping Age, 33:611-613 (1928). G. Weinblum Zeit. Angew. Math. n. Mech. 16: 375-376 (1936), 18:122-127 (1938).

[21] E. A. Sperry, Naut. Gaz. 113:767-776 (1927), R. W. Crowley, Motorship, 13: 772-773, 928-933 (1928), N. Watanabe, Math. Phys. So. Tokyo, Proc. 9:369-387 (1918).

[22] Lawrence Spear, Trans. So. Naval Arch. and Mar. Eng. (1898).

[23] G. H. Bryan, Engineering, 67:729-731 (1900), Nature, 62:186-188 (1900).

5. *The development of experimental methods.* At Norwich in the year 1868 the attention of the British Association for the Advancement of Science[24] was drawn to the deficiency of existing knowledge of the stability, propulsion and sea-going properties of ships, and to the need of further experiments on those subjects as a basis for the extension of theoretical investigation. A Committee was accordingly formed with C. W. Merrifield as chairman. It included four other members of the Royal Society of London, namely G. P. Bidder, Captain Douglas Galton, Sir Francis Galton and Professor W. McQuorn Rankine the distinguished mathematical engineer. The last member of the Committee was Mr. William Froude who had carried out private experiments with model ships on ponds, rivers, and lakes. In 1838 he had taken part in an investigation on the frictional resistance of ships for the shipbuilding firm of Brunel and this experience was probably of much influence in the development of the train of thought which culminated in the idea that by separating the frictional resistance from the total resistance, a residual resistance would be obtained which was to be attributed largely to the formation of waves on the surface of the water and so would depend upon gravity. If, then, models were tested in a tank, a law of comparison might be applied to the residual resistance and lead to an estimate of the residual drag for a full sized ship. Froude accordingly submitted a plan for model experiments to the Committee of the British Association and to the British Admiralty.

Froude was overruled by the other members of the said Commtitee who in a letter to the British Admiralty recommended experiments upon full-sized ships in the fiords of Norway or in the inland water of the west coast of Scotland. It was stated that the Committee did not wish by any means to discourage experiments made by means of models, which can evidently be had in greater number and in larger variety at much less expense than on full scale. Froude's reasons for opposing the plan for full scale tests were, moreover, added to the report. A copy of this report was sent also to the Institution of Engineers in Scotland whose Committee gave a general approval of the recommendations, particularly the proposed experiments and those on rolling. This Committee also called attention to the French Government's method of experimenting in which a ship was set in motion at a high speed and then allowed to lose its speed gradually with power shut off in the hope that an estimate of the resistance of the water and of the added mass could be derived from observations of the rate of retardation. The idea of added mass had been more or less

[24] British Association Report (1870).

familiar since the time of DuBuat's experiments on pendulums swinging in water[25] and the calculations of Poisson, Green and others. For a good account of the early work reference may be made to the memoir of Sir George Stokes.[26] For the experimental determination of the added mass in the case of a ship reference may be made to papers by Froude,[27] Abell,[28] Browne, Moullin and Perkins[29] and by Lewis.[30]

6. *Model Basins.* The British Admiralty decided in favor of preliminary experiments upon models to be conducted by Mr. Froude but did not give any reason for rejection of the proposal to experiment with Her Majesty's ships. It is likely that shipbuilders were at that time more interested in the problems of propulsion than in those of stability. Also Froude's plan of making tests under steady conditions seemed likely to give results that would be directly useful in design.

Froude had already constructed a small tank in his garden at Paignton in Devonshire but on moving a short distance to a new home at Chelston Cross, Cockington (near Torquay) he constructed a parallel-sided tank 278 feet long, 36 feet broad at the top and 10 feet deep.[31] In this he towed, with his dynamometric apparatus, thin boards of wood so as to determine the law of surface drag. This law was then used to find the surface drag of a ship's model, the area of the wetted surface of the model being used in place of the area of the wetted surface of the board. The influence of the ship's curvature was thus assumed to be small. This assumption is fairly good for slender ships of normal shape and for airships but is not applicable to the hull of a seaplane. Some idea of the effect of curvature may be derived from a mathematical investigation by C. B. Millikan[32] in which the boundary layer theories of Prandtl and von Karman have been extended so as to be applicable to bodies of revolution. The analysis involves an approximate solution of the fundamental differential equation and some plausible assumptions for the turbulent portion of the boundary layer. The results have been used to form an estimate of the surface drag of a submarine. Amtsberg remarks that the measured form drag for the fullest of a set of cigar shaped models was about 10% of the total drag and that the excess of the

[25] Chevalier DuBuat "Principes d'Hydraulique" (1782).
[26] G. G. Stokes, *loc. cit.* Reference (9).
[27] W. Froude, Trans. Inst. Naval Arch. 15:36 (1874).
[28] T. B. Abell, Engineering, 130:154-156 (1874).
[29] A. B. Browne, E. B. Moullin and A. J. Perkins, Cambridge Phil. So. Proc. 26: 258-272 (1930).
[30] F. M. Lewis, Jn. So. Naval Arch. and Mar. Eng. 37:1-18, disc. 18-20 (1930).
[31] Engineering, 136:60 (1938).
[32] Trans. Am. So. Mech. Eng. (APM-54-3, 54:29-43 (1933)).

total drag over the flat-plate skin friction value ranged from 2% to 5% with increasing fulness. According to Millikan's theory the skin friction for these models should have been from 5% to 6% in excess of the flat plate value.

With the aid of the British Admiralty, Froude then measured the drag of a model of the steamship Greyhound and used his law of comparison to compare the residual drag with the residual drag found from full scale tests. The results were fairly satisfactory and were regarded as illustrative of the value of model tests. In the full scale tests the French method was adopted to some extent.

A second tank was now constructed by the Admiralty at Haslar near Portsmouth and placed under the direction of R. E. Froude, son of William Froude. Shipbuilders also began to take notice and in 1883 the enterprising firm of Denny Brothers of Glasgow built an experiment tank at Dumbarton at the instigation of William Denny. Besides the usual tests experiments were made to find the effect on a ship of artificial waves. These were produced at first by means of a manually operated board but an apparatus for producing waves is now standard equipment in many marine laboratories.

American engineers were interested in the work of the model tanks[33] long before an appropriation for the construction of one was granted by Congress, largely through the efforts of Congressman Hilborn. A basin was constructed in 1898 in the S. E. corner of the Washington Navy Yard in a building 500 feet long, the tank itself being of length 350 feet and larger than any at that time in existence. The special machinery and apparatus were installed the following year and Naval Constructor David W. Taylor (now Admiral Taylor) was placed in charge.[34] He did such adimrable work on the resistance and propulsion of ships that the new model basin which has been constructed at Carderock in Maryland[35] has been named the David W. Taylor Model Basin. The specifications for the equipment for this basin indicate how much thought and care have been used in the design and the extent to which model testing has been developed.[36] In some cases models are self propelled.

There are now at least 27 model basins in existence, the largest being in this country according to information at present available. At Langley Field there is a basin 2000 feet long in which speeds of 58 miles an hour have been obtained and perhaps exceeded. At the

[33] W. F. Durand, Marine Engineering, Nov. (1897).
[34] D. W. Taylor, Am. So. Naval Arch. and Marine Eng. (1900).
[35] P. Ruhle, Marine Eng. and Shipbuilding Rev. 42:486-487 (1937), H. C. Fischer, Jn. Am. Concrete Inst., 10:317-336 (1939).
[36] Capt. H. B. Saunders, Engineering, 151:125 (1941). Trans. So. Naval Arch. and Mar. Eng. 46:307-320, dis. 320-324 (1938), Advance paper Nov. 14-15 (1940).

University of Michigan there is a tank 300 feet long, and there is also a tank at Stevens Institute of Technology.[37] Shipbuilding Companies in this country have made much use of the results of tank experiments.

Of the tanks in foreign countries special mention should be made of the large tanks at Leningrad, Hamburg, Rome, and Teddington (near London). The first of these has a length of 1810 feet and has an equipment permitting a speed for the model of 44 miles an hour. One tank at Hamburg has a length of 1148 feet while that at Rome has a length of 680 feet. The tank at Teddington has been named for William Froude and was intended originally for use in the improvement of the design of mercantile vessels, it has a length of 680 feet and was donated by Sir Alfred Yarrow.

There are also model basins at Greville (Paris), Berlin, Bremen, Vienna, Moscow, Spezia (Italy), Wageningen (Holland), Trondhjem (Norway), Hoegskolens shipbuilding laboratory (Sweden) and at Teishinsho (Japan).

Shortly before the war a plan of co-operation between the national ship testing establishments in England, Italy, Germany, and Holland was initiated for the comparison of a series of resistance and self-propulsion tests on models made to a given series of lines. The tests at the Froude tank and the Dutch tank were completed for models of both fast and slow vessels. The results indicated that the resistances measured at Teddington are about 4% higher than those measured in Holland. Besides the effect of possible differences in turbulence there are small differences due to the finite depth and breadth of the channels. Some estimates have been made by G. Weinblum,[38] on the basis of an expression obtained by Sretensky,[39] for the wave resistance of ships in channels of finite depth and width. It seems that for the usual sizes of model and channel the side walls hardly produce any appreciable disturbances; but these can arise when the length of the model is equal, say, to the breadth of the tank.

[37] L. J. Hooper, Trans. Am. So. Mech. Eng. (HYD- 1-3) 58:577-586 (1936).

[38] G. Weinblum, Zeit. angew. Math. u. Mech. 17:365 (1937). Schiffbautechnischen Ges., Jahrbuch 39:266-289, disc. 289-291 (1938).

[39] L. N. Srettensky, Phil. Mag. (7) 22:1005-1013 (1936), C. R. (Doklady) de l'Akad. des Sc. USSR (2) 7:265-267 (1936).

A former teacher in the Philippine Normal school, Dr. Franklin Bobbitt was born in English, Ind., in 1876. He received the Bachelor's degree at Indiana university in 1901, and the Ph.D. degree at Clark unibersity in 1909. He will continue his work on a comprehensive volume "The Curriculum of Modern Education" at his home in Waldron, Ind,

The Teacher's Department

Edited by
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EDUCATIONAL INTERESTS OF TEACHERS OF COLLEGE MATHEMATICS

The teacher of college mathematics is primarily a teacher and secondarily a research mathematician. As a teacher, his educational interests manifest themselves both in the classroom and outside the classroom.

In his official duties as a classroom teacher, the teacher of college mathematics is, or should be, interested in educational problems dealing with instructional aims of mathematics, instructional materials, instructional methods, evaluation of instruction, and psychological principles inherent in mathematical instruction. In his unofficial contacts outside the classroom, he has conferences with colleagues, administrators, students and perhaps parents, attends educational meetings, subscribes to journals, and is a member of learned societies. In both capacities, his educational interests manifest themselves either in the contributions he makes to the formulation or solution of instructional problems, or by his reactions to the reported contributions of the other teachers.

As intimated above, a small number of college teachers of mathematics are actually engaged in "pure research", that is, in research in the fields of mathematics whose subject matter is presented at the graduate level, if presented at all for instructional purposes. While not questioning the significance of such research, we wonder if the mathematical journals which are devoted to the promulgation of interest in collegiate mathematics have paid sufficient attention to instructional problems commensurate with the proportion of their subscribers who are primarily teachers.

The Teachers' Department of the NATIONAL MATHEMATICS MAGAZINE has been organized to fill such a need. It is planned to serve the teachers of mathematics specifically by accepting for publication: (1) reports of educational experiments conducted by departments of mathematics or by individual teachers of mathematics; (2) discussion of educational problems dealing with aims, materials,

methods, or evaluation of mathematical instruction; (3) formulation or solution of instructional problems; (4) contributions concerning the psychological principles involved in mathematical instruction; and (5) historical as well as philosophical papers dealing with the development, nature, and manifestation of mathematical concepts and processes.

Any plan of action is desirable when its aims are worthy. The proposed plan can be carried out and its aims attained, if the teachers of college mathematics will cooperate with the editors of the Teachers' Department: (1) by contributing papers and articles for publication; and (2) by inviting their colleagues to become subscribers and contributors.

J. S. GEORGES.

Types of Learning Products of Evaluation of Instruction

By J. S. GEORGES

The problem of evaluation is essentially a problem of measurement. Measurement presupposes quantitative aspects of the phenomenon. Qualitative aspects may be graded by means of comparison, but cannot be measured unless reduced to equivalent quantitative properties. While we may distinguish between a good instructor and a poor instructor, we cannot distinguish between two good instructors unless we reduce their goodness of instruction to quantitative terms. Similarly, while for the purpose of records and of transferring of credits marks of S denoting superior and F denoting failure may be adequate, yet for the purposes of evaluating the learner, in so far as learning is concerned, such marks are utterly meaningless. Does F mean zero learning? If not, what does it mean? Does S mean maximum learning? Is maximum learning absolute or relative?

Considered then as a measurement problem, evaluation of instruction must be analyzed in terms of its inherent quantitative aspects. In certain types of instruction quantitative data are not relevant and are, on the basis of such instruction, meaningless. If, for example, we define appreciation of art or of music as a gratification of the esthetic needs of man, we are unable to measure appreciation in terms of any quantitative units. It is quite true that works of art can be evaluated in terms of certain arbitrarily defined units of esthetic measure, yet

such units fail to measure our appreciation of them as art. But even in those domains of instruction, whenever evaluation is attempted, some quantitative measures are employed.

Quantitative measurement, whatever may be its nature, presupposes a unit of measure and a process, direct or indirect, of applying that unit of measure. Thus, for example, distance may be measured directly or indirectly in terms of certain standard units of length. Time may be measured indirectly in terms of units of time by means of arc units of a circle. Speed may be measured indirectly in terms of derived units of length and time. Similarly, with many other types of measurements which are familiar to us. But how shall we measure instruction? What units shall we use? What measuring instruments shall we employ? Clearly, we cannot select a universal unit which will measure different types of instruction. A unit that measures ability of performance cannot be adequate to measure understanding. A unit that measures recognition will fail to measure ability.

Consequently, for the purposes of evaluation, we must distinguish between the various types of learning products. Failure to recognize this fundamental principle has resulted in misrepresentation of the learning process. For example, many authorities represent graphically the curve of learning as having negative acceleration, that is, of being logarithmic in nature. Does that mean that if understanding is plotted as an ordinate and time or effort as an abscissa, the curve, if any, will be logarithmic? Far from it! It can be demonstrated that even in the case of recognitions, the learning is sequential or serial, and it is a function of an integral variable, that is, having a discrete and not a continuous range. And when we attempt to apply the logarithmic curve to appreciation type of learning, it does not make any sense at all. The only type of learning which may be assumed to follow the pattern of the logarithmic curve is perhaps the performance or ability type. But even in this case perhaps the arctangent curve or the "Gudermannian", that is, arctangent hyperbolic sine, is a better approximation.

For the purposes of evaluation we classify the learning products into four arbitrarily chosen types, namely, recognitions, understandings, abilities, and appreciations. Even though these four types of learnings are distinct, yet they do overlap and one type may be associated functionally with another type. Thus we may have recognition without understanding. We may have understanding without appreciation. We may also have recognition followed by understandings, and understanding together with the associated abilities.

In its simplest form learning consists of recognition, that is, of identification of an entity. The entity may be an object, a process, a

concept, or any fact whatsoever. If we represent an entity symbolically by x, its recognition depends upon its identification as belonging to a class C_x, that is, a class of similar x's. This recognition may be the result of one or more properties possessed by x. Symbolically we may write x^P. It is in terms of the property P, which is associated with the entity x, that similar x's are grouped together into the class C_x.

Much of the factual information obtained in our classes of instruction is of recognition-type. In the case of physical entities, the property P may designate magnitude, or texture, or color, or substance, or any other physical property. In the case of conceptual entities, the property P may designate meaning, or validity, or perceptual manifestation, or logical definition, or any other conceptual property. In the case of a process, the property P may designate a definition, or law, or result, or any other manifestation of the process.

Many examples of the recognition-type of learning can be presented from different fields. For example, one hears a piece of music, and recognizes it as a symphony; and may further recognize it as being Schubert's Unfinished Symphony. One sees a curve and recognizes it as a spiral, and may further recognize it as being the spiral of Archimedes. One reads a statement concerning terminal velocity and recognizes it as the velocity of a falling body affected by air resistance. One is told that sulfanilamide is a powerful drug in checking pneumonia, and recognizes the truth of the statement by believing it. A multitude of other examples might be added to the list illustrating the recognition-type of learning.

When we attempt to evaluate this type of learning in our classes, we encounter at once the obvious fact that many different factors affect the process of learning. Apperceptive mass plays an important part, for new facts are learned and interpreted on the bases of older facts of a similar nature in the possession of the learner. Memory plays an important role, for the retention of the acquired facts depends upon the memorizing powers of the learner. The method of instruction plays an important role, for by one method the process of learning may be accelerated to a greater degree than by another method. The nature of the subject plays an equally important role, for one subject lends itself more readily to memorization than another subject. The learning powers, interests, and applications of the learner play a very significant role, for upon them in the final analysis depends the process of learning.

Acquired recognitions may be independent or related. By independent recognitions we mean a sequence of recognitions each of which is acquired independently of the others. Such a sequence may be

represented symbolically by R_i for $i=1,2,3,\cdots, n$. By related recognitions we mean a series of recognitions each of which is acquired as a consequence of other previously acquired recognitions. Such a series may be represented symbolically by S_i, where $S_i=\Sigma R_i$ for $i=1,2,3,\cdots, n$.

For example, the following recognitions constitute a sequence of independent facts. R_1 : 5 and 7 are integral numbers. S_2 : 5/7 is a rational number. Here $S_2=R_1\oplus R_2$, in which $R_1\equiv 5$ and 7 are integral numbers; $R_2\equiv$ the symbol $(-)$ written between two integers in the designated manner means their ratio; and $\oplus$ means "taken together."

The process of evaluation of the recognition-type of instruction will be presented in detail because the process can be applied with modifications to the other types of instruction.

The first step in the evaluation is the identification and listing or classifying the many recognition-type of learning products which constitute the instructional aims or objectives of the unit of instruction or of the course. It may be true that not all of these learning products will be attained by all of the students. But that is the very problem which evaluation is intended to solve. It may equally be true that the instructor may not be successful in teaching all of those learning products to the extent of actual learning by the students. Nevertheless such a list is essential to a scientific procedure in the evaluation of instruction.

The second step in the evaluation of instruction is the selection of a representative sample. In the language of the statistician the prepared list of learning products may be called a population. Random sampling consists of the selection of a small sample out of the population which is examined to determine the characteristics of the entire population. One sometimes wonders if the procedure employed in our schools of subjecting a student to answer three hundred questions in a period of two hours is sound when analyzed on the basis of the theory of random sampling.

The third step consists in the formulation of a test based upon the items of the selected sample. In the objective type of test, the best forms which lend themselves readily to the evaluation of the recognition-type of learning are the multiple choice and the matching; although the true and false form and the completion form may also be used in some cases.

The fourth step consists in the weighing of the various items of the test. While such weights are arbitrarily chosen, their judicious use will reduce materially the work involved in the computation of the various measures used in the analysis of test results. For the purposes of comparison and of determining the validity of the test the

same total weight may be used for each test and for succeeding semesters. In the department of mathematics at Wright Junior College we have been using the total weight of 60 for each test for the past seven years. Sixty was chosen because of its divisibility by a large number of integers.

The fifth step consists of the determination of the reliability of the test, that is, of the selected sample as truly representing the stated list of objectives. The reliability may be measured in terms of the probable error of the mean. An approximate formula for the probable error of the mean is

$$e=0.6745\sigma/\sqrt{n}$$

where σ is the standard deviation and n is the number of items in the sample. The interpretation of the probable error is as follows. The chances are even that the mean of the entire population will lie between $\bar{x}-e$ and $\bar{x}+e$, where $\bar{x}$ is the mean of the sample.

The final step is the analysis of the test results. While such an analysis depends upon the purposes for which it is made and the time involved in making the analysis, nevertheless, if the test is to be used for the purposes of evaluating instruction and not merely giving the student a mark, then an analysis along the following lines is extremely useful.

If more than one class is being tested on the same unit, or course, equivalent forms of the same test are administered the same day. The mean or the median for each class is computed. The discrepancy between the means for the various classes can be studied to advantage. Such discrepancies reveal a great deal. Incidentally, the mean may be used to transform the responses on the test to marks. The use of standard deviation in the analysis of the fluctuation of the individual responses from the mean is recommended.

Index numbers can be used to determine the efficiency of instruction as well as of learning. The following index number has been found quite useful in this connection. Let c be the number of correct responses of the class on any particular item, f the number of incorrect responses, and n, the total number of responses. Then the index number I is

$$I=(c-f)/n.$$

Obviously, the values of I range from -1 to $+1$. $I=-1$, when c, the number of correct responses, is 0. A negative index number for any particular item shows inadequate instruction and unsatisfactory learning. Furthermore, if the forms of the same test are used in dif-

ferent classes, a negative index for instructor x contrasted to positive indexes for the other instructors, again reveals a great deal.

Obviously, index numbers can be used in measuring the validity of the test, in revealing whether or not an item is a misfire item, in determining whether or not a given learning product can be attained at that level, and many other phases of instruction.

The second type of learning, as pointed out above, consists of understanding. In its simplest form understanding is manifested whenever a concept or a process is rationalized. Since understanding is usually associated on the one hand with recognition and on the other hand with ability, many authorities attempt to depict understanding as a point on the curve of learning, whatsoever its nature, whose ordinate represents the level of understanding. According to this theory, actual learning may take place without attaining the level of understanding. Moreover, after the level of understanding has been reached, learning may continue in terms of additional recognitions or in terms of more efficient skills.

As in the case of recognitions, understandings may be independent or related. The process of learning consisting of independent understandings is sequential, while that consisting of related understandings is serial. Essentially, for the purposes of evaluation, a single understanding, whether it is in a sequence of independent understandings or is a series of related understandings, may be considered as that type of learning which investigates the conditions under which a stated fact will occur, the reasons for its occurence, and the manner of its manifestation. By recognition, the entity x is identified as belonging to the class C_x by means of the class property P. By understanding the existence of the entity x is justified, its inclusion in the class C_x is established, and the nature of the class property P is explained. Symbolically, we may represent the idea as follows: $C^P \cdot x^P \cdot) \cdot C_x$. The symbolic statement means there exists a class C which contains all the elements possessing the class property P. There exists an entity x possessing the property P. It is concluded that x is an element belonging to the class C.

One type of understanding deals with properties of entities. It may proceed by induction and generalization to establish the existence of the class C_x, as a logical conclusion from the observations that each of the various x_i for $i=1,2,3,\cdots,n$ has the property P. It may also demonstrate by deduction that property P belongs to the entity x due to the fact that x is contained in the class C_x all of whose elements possess the stated property P.

Consider, for example, the statement: Franklin Delano Roosevelt is a native born American. The understanding of the truth of the

statement by a person not having any knowledge of the birth place of Franklin Delano Roosevelt manifests itself as follows: R_1 (recognition) Franklin Delano Roosevelt is the President of the United States. R_2 (recognition) the president of the United States is a native born citizen. U (understanding) Franklin Delano Roosevelt is a native born citizen.

Consider, as a second example, the statement: A molecule of water consists by volume of two parts hydrogen and one part oxygen, that is, symbolically, H_2O. U_1 (understanding) or R_1 (recognition): water can be decomposed by electrolysis into its constituent elements. U_2 or R_2: the elements obtained by the electrolysis of water are only hydrogen and oxygen. U_3 or R_3: when water is decomposed by electrolysis, the volume of hydrogen liberated into one tube is twice the volume of oxygen liberated into the other tube. U_4: A molecule of water consists of two atoms of hydrogen and one of oxygen.

As a third example consider the statement: $\sqrt{2}$ is not a rational number. R_1 (recognition): A rational number can be reduced to the form a/b, where a and b are integers with no common factor. R_2 or U_2 (understanding): The square of a rational number is rational. R_3 or U_3: the square of $\sqrt{2}$ is 2. U_4: If $\sqrt{2}=a/b$ then $2=a^2/b^2$. U_5: If $a^2/b^2=2$, then $a^2=2b^2$. U_6: Since $2b^2$ is even, then a^2 is even. U_7: If a^2 is even then a is even. U_8: If a^2 is even it contains the factor 4. U_9: Then b^2 is even: U_{10}: If b^2 is even then b is even. U_{11}: If a and b are both even, they have the common factor 2. U_{12} or R_4: Of two mutually exclusive events either one or the other but not both are true. U_{13}: therefore $\sqrt{2}\neq a/b$.

Another type of understanding deals with relationships between two or more entities belonging to the same class or to different classes. In this form we may express the learning symbolically as xRy, which means that the entity x is associated with the entity y according to the relationship R. While recognition may identify the relationship and accept or deny its existence, understanding, on the other hand, establishes the existence of the relationship by considering the causes which produce the stated relationship. As in the case of properties of entities, the understanding of the individual fact xRy may be a logical consequence of the deduction from the three facts CRC', C_x, and C_y. That is, every element of the class C is associated with a corresponding element of the class C', the element x is in the class C and the element y is in the class C'. Therefore x is associated with y according to the relationship R.

The learning may also proceed on the basis of induction by generalizing the separate facts x_iRy_i into C_xRC_y. This is the procedure in the establishment of empirical laws or theories in sciences.

The understanding-type of learning is peculiar to logical systems, and to sciences. Other subjects may present situations in which rationalization is attained. To that extent such subjects become scientific in nature if not in practice.

Evaluation of understanding as a learning product presents a more difficult problem than the evaluation of recognition. This difficulty is due to two main factors. The first is the inability of the evaluator to distinguish between recognition and understanding. Many an instructor confuses memory with understanding. The second factor causing difficulty in evaluating understanding is the inadequacy of the evaluation instrumentality, such as tests, to present situations which call for the exercise of understanding. Most of the objective type tests using the standard forms are designed to test recall of facts. Recalling facts is not understanding.

Many investigators in the field of evaluation have recommended that a test of understanding should present situations in the form of problems in which understandings are required to obtain solutions. One objection to the problem-type of evaluation is the fact that problem solving requires not only understanding but also ability or skill. We must recognize the fact that one may have an understanding of a process without being able to use it in a problem-situation. This is especially true in the sciences and mathematics, which, as stated above, are primarily understanding-type of learnings. To illustrate this objection, consider the following familiar examples. One may understand that a certain cubic equation has three real roots yet be unable to determine the three roots. One may understand that the differential equation of the motion of a body falling under the force of gravity and under a force opposing motion (such as the air resistence) and proportional to the velocity at any instant is $d^2y/dx^2 = g - kv$, yet be unable to solve the differential equation. One may have an understanding of the theory of evaluation yet not be able to demonstrate its applications in a given special case. One may understand, as one critic has put it, that Schubert's Unfinished Symphony is a mirror of Schubert's own sorrowful life, "likening its occasional moments of exuberance in the midst of its sober passages to its composer's career, where handfuls of good fortune were thrown in the bushels of adversity", yet be not able to play a single melodic line.

The second objection to the problem-type of evaluation of understanding is the common practice of evaluators to disregard the sequence of steps required in the solution of the problem and to consider only the last step, namely, the answer. This is a malpractice which defeats the purpose of evaluation and its justification by its users has

created a pathological condition in the mind of the learner who confuses getting an answer to a problem by any method whatsoever as a criterior of actual learning. This being an abuse of the method, it can be remedied and the method can be presented in its true merit.

In the unit tests administered by the Department of Mathematics at Wright Junior College the second part of each test consists of items based upon understanding of the concepts and processes of the unit. The random sampling method is used so that only a few, say ten, items are chosen. Then each of the items is analyzed as to whether it constitutes a single understanding, or a composite understanding, and if composite, whether it is sequential or serial. The test form best adapted to the evaluation of understanding is the completion form. For example, the item may be based upon the understanding of the fact that the rate of change of an exponential function, such as the population curve, is proportional to the value of the function. The test item may consist of the following statement and the anlyzed steps. The speed v of a chemical reaction increased with temperature t at a rate which at any temperature was 4% of the speed at that temperature.

(a) The function representing the stated relationship is. . . . .(exponential)
(b) The formula is of the form. . . . .($v = v_0e^{rt}$)
(c) The stated rate of 4% is the value of. . . . .(r)
(d) If the initial speed was 10 units, the formula reduces to. . . . .($v = 10e^{.04t}$)
(e) When the temperature was increased 20°, the speed was. . . . .($10e^{.4}$)

The evaluation of the answer $10e^{.4}$ calls for a certain ability in the use of logarithms which is not a part of the understanding called for in the item.

The third type of learning consists of the development of certain abilities and acquisition of specialized skills. The instruction seeking to develop manipulative skills may or may not aim at the learning of the associated recognitions and understandings. In certain subjects where the primary aim is the development of skills, little or no attempt is made to attain understanding. In other subjects, where the application of a process requires special kind of manipulative abilities, these abilities may be acquired with or without the rationalization of the process.

Since this type of learning is primarily one of performance, its evaluation may be on the basis of determining whether or not the individual is able to perform the task which calls for the use of acquired skills. Very few types of learnings are of this nature. The individual may not perform the task as an expert would, and may make mistakes, but he does perform it to some extent. In other words, the test may

be based upon the determination of the degree of accuracy in the performance. Many of the so-called standardized tests and scales in arithmetic and algebra are of this nature.

The evaluation may also be based upon the rate with which the task is performed. Where no attempt is made to take into account the associated recognitions or understandings, which do influence the rate of performance, timed tests, as they are called, are quite adequate. The main objection to timed tests is the variance in both the rate of learning and the rate of performance. Two individuals may not learn with equal rates, due to apperceptive mass, inherent abilities, interests, applications, and many other causes. This calls for different rates of performance. Timed tests do not consider individual differences. Even in the standardized types, the established forms do not enable the determination of a satisfactory time factor for a given individual.

The type of instruction which aims to develop special abilities must of necessity provide opportunity for practice. However, in many fields of instruction we must distinguish between the type of learning which acquires skills by imitation and the type which develops abilities on the basis of a rationalization of the process. The one is learning by rote and the other is rationalization of the process. By the one method the individual learns to do, but he does not know why he does it that way. By the second method also he learns to do, but he knows what he is doing and why he does it that way and not some other way.

In the first type, drill material may be used to determine both the rate of performance and the accuracy, such as standardized tests or scales in mathematics, timed tests in typing, and other tests devised to measure speed and accuracy.

In the second type, the test items must take into consideration the recognitions or understandings with which the given ability is associated. In this type of test the element of time is not as important as in the former type. The test item calls for the performance of a certain process. The student either does or does not perform the process and if he does, then the accuracy of his performance is evaluated. Of necessity, the test item, if based upon a problem, should provide for testing the recognitions or understandings associated with the ability. For example, consider the following problem. Two forces of 75 pounds and 50 pounds respectively act upon a body at an angle of 30° with each other. Find their resultant and its direction. The particular ability called for in the solution of this problem depends upon the following recognitions or understandings. R_1 (or U_1): Either force may be represented by a horizontal vector. R_2 (or U_2): The direction of the other vector is at an angle of 30° with the horizontal.

R_3 (or U_3): The two vectors form the sides of a parallelogram of forces. R_4 (or U_4): The diagonal of the parallelogram represents the resultant. R_5 (or U_5): The magnitude of the resultant is given by the law of cosines: A_1 (Ability): Substitution of 50, 75, and 30° in the formula for the Law of Cosines. A_2: Solution of the equation $r^2=50^2+75^2+2\times 50\times 75\cos 30°$. A_{21}: Evaluating 50^2. A_{22}: Evaluating 75^2. A_{23}: Evaluating $2\times 50\times 75\cos 30°$. A_3: Evaluating r^2. A_4: Evaluating r. Similar steps are involved in the solution for the direction of the resultant. Without such an analysis, it is impossible to determine what particular step or steps were responsible for an incorrect solution of the problem.

The fourth type of learning consists of the establishment of certain adaptations which we call appreciations. As pointed out above appreciations may result from understandings. But it is not true that understandings always result in appreciations. Furthermore, the acquisition of certain specialized abilities may result in the appreciation of the process upon which the abilities are based. But again it is not true that abilities always develop appreciations. Thus appreciations may be independent of understandings and abilities, or may be dependent upon them, or closely associated with them.

If, as some have defined them, appreciations consist of gratifications of esthetic needs of man, many different fields offer opportunities for acquiring such appreciations. Music, literature, fine arts are the fields where the primary educational aims are the acquisition of appreciations. However, many other subjects, especially the sciences, also offer opportunities for the development of appreciations. In the former case, the appreciations take the form of intensifying the desire for beauty, for harmony, for order and at the same time of satisfying that desire in a measure. The esthetic capacity which is an invariant of man may seek expression in many different ways. A poem, a painting, or a symphony may give an expression to an esthetic feeling that the individual reading the poem, looking at the painting, or hearing the symphony has. Often such feelings cannot even be analyzed. The individual says he likes the poem, or the painting, or the symphony, often without being able to explain why he likes them, and when such explanations are given, often they are not entirely too convincing. The individual reacts favorably, and that is, in a measure, the best measure of his appreciation.

Appreciations in sciences and mathematics, though primarily of a different nature, are akin to the appreciations in fine arts. One may appreciate the simplicity, the compactness, and the power of a formula. One may see rhythm in an infinite sequence. One may feel that there

is beauty of form in an algebraic expression in the same sense that the beauty of form of a geometric figure appeals to him.

However, when we consider the problem of evaluating appreciation, whatever may be its nature, we encounter extreme difficulty. How may we reduce to a quantitative measure the esthetic gratifications resulting from instruction? We might measure to some extent the degree or intensity of such gratification. But what does such a measure mean? One responds according to his inner esthetic capacity. These capacities differ in different individuals, hence we expect different grades of responses, and the one is as significant as the other.

In the sciences, very little attempt has been made to evaluate appreciations directly. Perhaps the best test is the so-called functional test, but such a test cannot be administered properly in a class room.

The Bulletin of the California State Department of Education for June, 1941, is entitled *Some Mathematics Practices in California Secondary Schools*. This is the report of the subcommittee on mathematics of the general education committee of the Association of California Secondary School Principals. There are some forty pages in this report. Some of the main topics are: mathematics and general education, what California secondary schools are doing, what California mathematics teachers believe, and implications for future mathematics practices in secondary schools. This bulletin is a publication of the California State Department of Education Sacramento, California. Dr. C. C. Trillingham, Assistant Superint3ndent and Director of Secondary Education of Los Angeles, served as chairman of the committee. The members of the committee were: Harry G. Alway, Harold D. Aten, William W. Booth, Dale Carpenter, Bruce M. Casey, E. Howard Floyd, Robert S. Fraser, Edwin M. Hemmerling, Elmer M. Krehbiel, Robert B. Herrera, Donald W. Larwood, Earl Murray, Joseph O'Loughlin, John S. Reed, Frank B. Lindsay.

—Reported by L. J. Adams.

A member of the University of Chicago faculty for 33 years, Dr. Gilbert A. Bliss was born in Chicago in 1876. He was awarded the Bachelor's and Master's degree in 1897, and the Ph.D. degree in 1900 at the University of Chicago, and an honorary Doctor of Science degree at the University of Wisconsin in 1935. After teaching at the universities of Minnesota, Missouri, and Princeton he was appointed associate professor of mathematics at the University of Chicago in 1908, full professor in 1913, chairman of the department in 1927, and Martin A. Ryerson distinguished service professor in 1933.

In addition to his research in mathematical analysis, Dr. Bliss conducts a course in exterior ballistics—the science which sends a long-range shells truly to their mark. He was scientific expert of the range firing section of the U .S. Army in 1918. His published works include books on the calculus of variations, and the science of exterior ballistics. A member of many learned societies, both here and abroad, Dr. Bliss is past president of the American Mathematical Society.

Problem Department

Edited by
ROBERT C. YATES and EMORY P. STARKE

This department solicits the proposal and solution of problems by its readers, whether subscribers or not. Problems leading to new results and opening new fields of interest are especially desired and, naturally, will be given preference over those to be found in ordinary textbooks. The contributor is asked to supply with his proposals any information that will assist the editors. It is desirable that manuscript be typewritten with double spacing. Send all communications to ROBERT C. YATES, Mathematics, L. S. U., Baton Rouge, Louisiana.

SOLUTIONS

No. 395. Proposed by *Paul D. Thomas*, Southwestern State College, Oklahoma.

Prove that the locus of the centers of all conicoids through the cubic curve $x=t$, $y=t^2$, $z=t^3$ is the surface $2x^3-3xy+z=0$, $(x=u,$ $y=v$, $z=3uv-2u^3)$. Find the asymptotic lines of this surface and show that their projections on the plane $x=0$ envelope the semicubical parabola $z^2=2y^3$.

Solution by the *Proposer*.

Since the given cubic curve lies on all three of the conicoids $xy-z=0$, $zx-y^2=0$, $x^2-y=0$, all the conicoids through the curve are represented by

$$f=a(xy-z)+b(zx-y^2)+c(x^2-y)=0.$$

$$f_x=ay+bz+2cx=0$$

$$f_y=ax-2by-c=0$$

$$f_z=-a+bx=0.$$

The elimination of a,b,c, among $f_x=0$, $f_y=0$, $f_z=0$ gives the surface $2x^3-3xy+z=0$.

The asymptotic lines are given by the differential equation

$$(1): \qquad D\cdot du^2+2D'dudv+D''dv^2=0$$

where D, D', D'' are respectively ΣXx_{uu}, ΣXx_{uv}, ΣXx_{vv} and X, Y, Z are respectively $(y_uz_v-y_vz_u)/H$, $(x_vz_u-x_uz_v)/H$, $(x_uy_v-x_vy_u)/H$. Using

the given parametric representation of the surface, it is found that $HD = -12u$, $HD' = 3$, $HD'' = 0$ and these quantities reduce (1) to $-2u\, du^2 + du dv = 0$. Integrating this we find

$$(2):\quad u = c_1, \quad \text{and} \quad (3):\quad v - u^2 = c_2$$

which are the equations to the asymptotics. The projection of (2) upon $x = 0$ is $S = 2c_1^3 - 3c_1 y + z = 0$. The elimination of c_1 between $S = 0$ and $S_{c_1} = 0$ gives for the envelope the semicubical parabola $z^2 = 2y^3$. The projection of (3) upon $x = 0$ is $T = y^3 + 3c_2 y^2 - 4c_2^3 - z^2 = 0$. The elimination of c_2 between $T = 0$ and $T_{c_2} = 0$ gives $z^2 = 0$ and $z^2 = 2y^3$.

No. 400. Proposed by *Howard D. Grossman*, New York City.

Prove that if a coin is tossed often enough, the chance approaches 1 that the number of heads will eventually at least once exceed the number of tails by 1. Does this prove that if you play long enough, you are practically certain to win (or equally certain to lose)?

Solution by *H. S. Grant*, Rutgers University.

Let $P(n)$ be the probability that the number of heads will exceed the number of tails by 1 at least once in n tosses. Then $Q(n) = 1 - P(n)$ is the probability that the number of heads will never exceed the number of tails in n trials. Let $N(t,h)$ be the number of cases of t tails and h heads after n tosses $(t + h = n)$ in which the event associated with $Q(n)$ is realized, and put

$$N(n) = \sum N(t,h).$$

Since, in enumerating $N(t,h)$ for $t + h = n$, we must proceed from those cases in which $t + h = n - 1$, and since, whether we toss a head or a tail, the probability is $\frac{1}{2}$; we conclude

$$Q(n) = N(n)/2^n.$$

We observe further that $N(2m) = 2N(2m - 1)$, for, in proceeding from a case in which $t + h$ is odd either a head or a tail is permissible. On the other hand, $N(2m+1) = 2N(2m) - N(m,m)$, since the cases associated with $N(m,m)$ would allow only a tail on the next toss. Thus we have

$$Q(2m) = Q(2m-1), \quad Q(2m+1) = Q(2m) - N(m,m)/2^{2m+1}$$

whence we derive

$$Q(2m-1) - Q(2m+1) = N(m,m)/2^{2m+1},$$

$$Q(1) - Q(2m+1) = \sum_{i=1}^{m} N(i,i)/2^{2i+1}.$$

Since $Q(1)$ is evidently $\frac{1}{2}$, we have

$$Q(2m+1)=\tfrac{1}{2}-\tfrac{1}{2}\sum_{i=1}^{m} N(i,i)/4^i.$$

The desired result, $\lim\limits_{n\to\infty} P(n)=1$,

will follow immediately when we have shown that

$$\lim_{m\to\infty} \sum_{i=1}^{m} N(i,i)/4^i=1.$$

To do this, notice that $N(i,i)$ is the same as the number of arrangements of i T's and i H's such that, in proceeding from left to right, the number of H's will never exceed the number of T's. By placing a T on the extreme left in any one of these arrangements we would obtain an arrangement of $(i+1)$ T's and i H's in which the number of T's would always exceed the number of H's in proceeding from left to right; and conversely, associated with each of these latter arrangements is one of the former, obtained by dropping the T on the extreme left. Thus $N(i,i)$ is obtained, for example, in T. C. Fry, *Probability and Its Engineering Uses*, Ex. 33, p. 78, in which we take $m=i+1$ and $n=i$. We find

$$N(i,i)=C_i^{2i+1}-2C_{i+1}^{2i}=C_i^{2i}/(i+1).$$

The desired summation may be found by the aid of C. Smith, *A Treatise on Algebra*, pp. 410-411, with $a=1$, $b=4$, $x=2$, whence we have

$$\sum_{i=1}^{m} N(i,i)/4^i=1-C_m^{2m+1}/4^m.$$

That the limit of this summation as $m\to\infty$ is unity is shown in the same book, p. 424, Ex. 2. Our proof is thus completed.

A negative answer must be given for the Proposer's question. All that is shown is that a player is practically certain sooner or later to have been ahead at least once. That is not the same as winning unless one of the players has and uses the privilege of stopping the play as soon as he is one point ahead, while his opponent has not also that privilege.

Editor's Note. Making use of the relations

$$N(t,h)=N(t-1,h)+N(t,h-1),\quad t>h$$

$$N(m,m)=N(m,m-1),$$

which are evident from the above discussion, the proof of the following formulas is easy by induction:

$$N(t,0)=1,\quad N(t,1)=C_1^{n-1},\quad N(t,2)=C_2^{n-1}-1,$$

$$N(t,h)=C_h^{n-1}-C_{h-2}^{n-1},\quad n=t+h,\quad 2<h<t.$$

We then deduce

$$N(m,m)=N(m,m-1)=C_{m-1}^{2m-2}-C_{m-3}^{2m-2}=C_m^{2m}/(m+1)$$

as found in the above solution.

No. 405. Proposed by *Robert C. Yates*, Louisiana State University.

Given the three distinct non-collinear points A, B, C. With the compasses alone, draw the circle inscribed to triangle ABC.

Solution by *D. L. MacKay*, Evander Childs High School, New York, N. Y.

By means of circles $A(AC)$ and $B(BC)$ obtain the symmetric C' of C with respect to AB and draw circles $C(CC')$, $C(CB)$, and $C'(C'A)$, the third cutting the first in D and the second in S. The circle $D(DC')$ cuts circle $C(CB)$ in W and circles $A(SW)$ and $B(SW)$ intersect in O, the circumcenter of ABC.*

Now draw circles $C(CO)$, $O(AC)$, $O(AB)$, $O(AC)$ cutting circles $A(SW)$ and $C(CO)$ in G and H respectively and circle $O(AB)$ cutting circles $A(SW)$ and $B(SW)$ in E and F respectively. Let U be one of the points of intersection of the equal circles $H(HA)$ and $D(DC)$, V one of the points of intersection of the equal circles $F(FA)$ and $E(EB)$. The one or the other of the circles $O(OU)$, $G(OU')$ will cut the circumcircle O in M and M', midpoints of the arcs having AC as common chord. The one or the other of the circles $E(OV)$, $F(OV)$ will cut the circumcircle in N and N', midpoints of the arcs subtended by AB. The circles $M(MC)$ and $N(NB)$ intersect at the incenter X.

To obtain the radius of the incircle, we take the symmetric X' of X with respect to BC and bisect XX' by means of circles $X'(X'X)$, $X(XX')$, intersecting at K, $K(KX')$ cutting $X'(X'X)$ in J, $J(JX')$ intersecting $X'(X'X)$ in X'', $X''(X''X)$ cutting $X(XX')$ in P, $P(PX)$ cutting $X'(X'X)$ in Q and $Q(QX'')$ intersecting $P(PX)$ in T, the point of tangency of the incircle with BC. The inscribed circle is $X(XT)$.

Essentially, this is the solution given by A. Q. Lanascol in his *Géométrie Du Compas*, 1925, p. 167.

Also solved by *Albert B. Farnell* and *Paul D. Thomas*.

*This implies that SW is the circumradius.—ED.

No. 407. Proposed by *Mathematics Club,* Tulane University.

Construct a square such that an interior point is distant 3, 4, and 5 units from three of its vertices.

Solution by *D. L. MacKay,* Evander Childs High School, New York, N. Y.

Let X be the required point within the square $ABCD$ whose vertices $A, B, C,$ are on circles $X(r_a)$, $X(r_b)$, $X(r_c)$, respectively. The problem then is to construct a triangle ABC similar to a given triangle $A'B'C'$ (here an isosceles right triangle) so that each of its vertices lies on one of three given concentric circles.

If X' is a point within triangle $A'B'C'$ homologous to X, then triangles $A'B'X'$ and $A'C'X'$ are respectively similar to triangles ABX and ACX and

$$(A'X')/(B'X')=r_a/r_b; \quad (A'X')/(C'X')=r_a/r_c.$$

Hence divide $A'B'$ internally and externally by D and E in the ratio r_a/r_b and $A'C'$ by F and G in the ratio r_a/r_c. The intersection of the circles on DE and FG as diameters is the point X'.

Take any point A on circle $X(r_a)$ and construct angles AXB and AXC equal respectively to angles $A'X'B'$ and $A'X'C'$ and obtain triangle ABC and thence the square $ABCD$.

In the general case the second point of intersection of the circles of proportional distances furnishes a second solution and, as A, B, or C can be on the larger circle, we may have twelve solutions.

G. Lamé treats the case of the equilateral triangle in his *Examen des Methodes,* 1818, p. 81. A solution of the general problem is given in Leybourne's *Mathematical Questions,* 1817, Vol. 1, p. 160.

Also solved by *H. T. R. Aude, W. B. Clarke,* and *P. D. Thomas.*

No. 408. Proposed by *Howard D. Grossman,* New York City.

Prove
$$1-\frac{1}{4}+\frac{1}{7}-\frac{1}{10}+\cdots=\frac{\log 2}{3}+\frac{\pi\sqrt{3}}{9}.$$

Solution by *Morris Chernofsky,* Yeshiva College, New York City.

$$1-\frac{1}{4}+\frac{1}{7}-\frac{1}{10}+\cdots=\int_0^1 (1-x^3+x^6-x^9+\cdots)dx$$

$$=\int_0^1 \frac{dx}{1+x^3}=\frac{1}{3}\log(1+x)$$

$$+\frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}-\frac{1}{6}\log(1-x+x^2)\Big]_0^1$$

$$=\frac{\log 2}{3}+\frac{\pi\sqrt{3}}{9}.$$

Also solved by *William N. Huff*.

No. 409. Proposed by *Paul D. Thomas*, Southwestern State College, Oklahoma.

Construct a triangle given the lengths of the median and internal bisector issued from the same vertex, and the angle between them.

Solution by *H. K. Humphrey*, Winnetka, Illinois.

Let AM be the given median, AH the bisector, and MAH the given angle. Draw MP perpendicular to MH intersecting AH extended in P. The circle whose center is on MP extended and passing through A and P is the circumcircle of the required triangle and cuts MH extended in B and C. Since MP is the perpendicular bisector of chord BC, it bisects arc BC (as does AHP) and thus ABC is the required triangle.

Almost identical problems result if, for the median, bisector, and altitude, all issued from the same vertex, there be given:

(a) the length of any one and the angles between it and the other two (3 cases);
(b) the lengths of any two and the angle between either and the third (6 cases);
(c) the lengths of all three.

Also solved by *Albert B. Farnell*, *D. L. MacKay*, *C. W. Trigg*, *Marian Wright*, and the *Proposer*.

No. 410. Proposed by *V. Thébault*, Tennie(France-Sarthé).

In what system of numeration, with base less than 100, is the three-digit number 333 a perfect square?

Solution by *D. C. Binneweg*, student, Colgate University.

Let b be the base and x^2 the required square, obtaining

$$x^2=3b^2+3b+3.$$

Solving this equation for b (using the denary scale) we have

$$b=(-3+\sqrt{12x^2-27})/6.$$

A necessary condition is that $12x^2-27$ be a perfect square, say y^2. The least positive solution of $y^2=12x^2-27$ is (3,9), whence all solutions are given by

$$y+x\sqrt{12}=(9+3\sqrt{12})(7+2\sqrt{12})^k, \quad k=0, 1, 2, \cdots .$$

$k=0$ yields $b=1$, which is of no use. $k=1$ yields the solution $b=22$. Since $k=2$ gives $b=313$, 22 is the only solution less than 100.

Also solved by *A. B. Farnell*, *C. W. Trigg*, and the *Proposer*.

PROPOSALS

No. 430 Proposed by *N. A. Court*, University of Oklahoma.

Four given spheres have a point E in common and intersect three by three in the points A', B', C', D'. The chords EA', EB', EC', ED' common to triads of given spheres meet the respective fourth given sphere in the points A'', B'', C'', D''. Prove that the segments $A'A''$, $B'B''$, $C'C''$, $D'D''$ are twice as long as the altitudes of the tetrahedron formed by the centers of the given spheres.

No. 431. Proposed by *E. P. Starke*, Rutgers University.

Show that it is possible to find a number such that, if its digits be written down in order twice, a perfect square is formed. What is the smallest possible number of digits?

No. 432. Proposed by *Paul D. Thomas*, Southeastern State College, Oklahoma.

Three concurrent lines through the vertices of triangle ABC meet the respective sides BC, AC, AB in the points A', B', C'. Circles are described on the segments AA', BB', CC' as diameters. The radical axis of the circle on AA' with the circumcircle of ABC meets BC in P_1. Similarly, using the other two circles with the circumcircle, points P_2 and P_3 are located on AC and AB. Prove that P_1, P_2, P_3 are collinear. A synthetic proof is desired.

No. 433. Proposed by *D. L. MacKay*, Evander Childs High School, New York.

Given circle (O), tangents PA and PB, point C on OA such that $BC=PB$, and chord BD bisecting angle PBC. Prove that the projection of BD on diameter AOE equals the radius of circle (O).

No. 434. Proposed by *Paul D. Thomas*, Southeastern State College, Oklahoma.

Given $$\lim_{x \to m} \log \sum_{n=0}^{\infty} a/x^n = 0.$$

Determine the relation between a and m.

No. 435. Proposed by *V. Thébault*, Tennie (France-Sarthé).

Given the tetrahedron $ABCD$ and a point P. The lines PA, PB, PC, PD, cut the faces BCD, CDA, DAB, ABC, in A', B', C', D'. Find the locus of points P such that the volume of $A'B'C'D'$ is constant. Discuss the case $(A'B'C'D') = 3(ABCD)$.

A member of the University of Chicago faculty for 32 years, Dr. Fred C. Koch was born in Chicago in 1876. He was graduated from high school in Oak Park, received the Bachelor's degree in 1899 and the Master's degree in 1900 from the University of Illinois. After two years as instructor in Chemistry at the University of Illinois he became a research chemist at Armour and Company. In 1909 he was appointed a fellow in bio-chemistry at the University of Chicago. He received the Ph.D. degree at the University in 1912, and became assistant professor of bio-chemistry in 1916, and professor and chairman of the department in 1924.

Among Dr. Koch's earlier investigations were the enzymes of the digestive tract; the gastric hormone, secretion, a hormone which influences pancreatic secretion, and the thyroid hormone. More recently he has been active in the field of vitamin D. He was among the first investigators to show that ultra-violet light converts cholesteral, a very widely dis-distributed wax-like substance, into vitamin D.

Dr. Koch's writings have been voluminous. He is a member of a score of learned and professional societies, including the American Medical Association and the American Academy for the Advancement of Science. Dr. Koch will remain at the University as Frank P. Hixon distinguished service professor emeritus of bio-chemistry.

Dr. Frederick S. Breed was born in Clarion County, Pa. He received the Bachelor's degree at Allegheny college in 1898, the Master's degree in 1905 and the Ph.D. degree in 1909 at Harvard university. He was appointed assistant professor of education at the University of Chicago in 1917 and became associate professor in 1926.

Author of books on the teaching of spelling and mathematics, and classroom organization and management, Dr. Breed has published studies of classroom size and teaching efficiency, validity of tests, classification of pupils, and theory of instruction. He was chairman of the Committee on Spelling of the American Educational Research Association in 1930-39.

Bibliography and Reviews

Edited by
H. A. SIMMONS and JOHN W. CELL

College Geometry. By Paul H. Daus. New York, Prentice-Hall, Inc., 1941. xv+200 pages. $2.50.

As stated by the author in the preface, the purpose of the book "is to introduce the student to a wide and extensive body of synthetic geometry." It seems to the reviewer that this purpose has been accomplished. In the wide range of material in the book most mathematics students should find something to interest and stimulate them. Students with a ready knowledge of their high school mathematics and trigonometry should be able to understand most of the theorems and do many of the exercises but, in general, a bit more mathematical maturity will be required for a complete understanding of all theorems and a thorough mastery of all the exercises. (A key to the exercises is provided with the wise caution to the reader to use it sparingly.)

The book is divided into the following eight chapters:

1. *Similar Figures.*
2. *Concurrency and Collinearity*
3. *Inversion.*
4. *Other Transformations.*
5. *Coaxal Circles.*
6. *Notable Points and Circles Connected with a Triangle.*
7. *Miquel Point and Simson Line.*
8. *Ruler and Compass Constructions.*

As one would expect from the chapter headings, such subjects are discussed as homothetic figures, theorems of Menelaus and Ceva, harmonic properties of the complete quadrangle and quadrilateral, the nine-point circle, Feuerbachs theorem, the problem of Apollonius, and many other famous problems and constructions associated with these. Interesting applications of inversion are made in proving Ptolemy's theorem and its extension and in inverting the Miquel Configuration. Pertinent facts on the configuration of the centers of a triangle are obtained through the use of Ceva's theorem. The connections between harmonic division, inversion, orthogonal circles, poles and polars, and coaxal systems are given careful attention.

The author has done well to emphasize the meaning and importance of necessary and sufficient conditions. Since many students may meet this terminology formally for the first time here it might have been well to give some homely illustrations of necessary and sufficient conditions. In theorems where special cases arise sufficient detailed explanation has been used to make the situation clear; e. g., the meaning of negative k in inversion, and the radical axis of two concentric circles. The need for postulating an ideal point is explained early in the text. Here too the exposition may be a bit too brief.

There is a large number of exercises, 173 to be exact, many of which should stimulate the student to independent reading and investigation. Drawings are good and not too numerous. The book should prove a valuable addition to the college text book literature of modern synthetic geometry.

University of Tennessee. J. A. COOLEY.

Mathematical Tables. By H. B. Dwight. New York, McGraw-Hill, 1941, 300 pages. $2.50

The collection of tables has been compiled by Professor Dwight, a teacher of Electrical Engineering at M. I. T., who has previously collected the *Tables of Integrals and Other Mathematical Data.*

The tables include five-place tables for all six trigonometric functions and the logarithms of four of them with argument in hundredths of degrees; sine, cosine, and tangent of thousandths of radians; $\sin^{-1}x$. $\cos^{-1}x$, and $\tan^{-1}x$ to fou rdecimals with $\Delta x=0.0001$. The tables for $\ln x$, e^x, e^{-x}, $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1}x$, $\cosh^{-1}x$, and $\tanh^{-1}x$ are to four deciamsls with $\Delta x=0.001$. Wherever possible in these first tables, the differences are listed in the right-hand column for ready interpolation.

Tables for advanced functions follow. These include the zonal harmonics, interpolation coefficients, elliptic integrals, Bernoulli and Euler numbers, Gamma function, Error function, Bessel functions, and Riemann-Zeta functions.

These tables are being used so frequently in certain engineering curricula that their compilation in a single volume is most welcome. The printing is by the offset or lithoprint method, which should reduce the number of errors to a minimum. In a few places in the advanced function tables there is too much reduction in size of print.

North Carolina State College. JOHN W. CELL.

Arithmetic in General Education. By Committee on Arithmetic National Council of Teachers of Mathematics, New York: Bureau of Publications, Teachers College, Columbia University, 1941. 335 pages. $1.25.

Arithmetic in General Education is the final report of the Committee on Arithmetic of the National Council of Teachers of Mathematics and is presented as a companion volume to the Tenth Yearbook, *The Teaching of Arithmetic.* It consists of fifteen chapters written by as many different authors, with topics such as the following included: *Arithmetic in the early grades from the point of view of interrelationships in the number system, A theory of instruction for the middle grades, Arithmetic in the senior high school, Subject matter in relation to personality, Grade placement, What becomes of drill? The social phase of arithmetic, The enrichment of the arithmetic course, Evaluation of learning, Recent trends in learning theory, Interpretation of research.* There is also a list of one hundred selected research studies related to educational practice, and another list of one hundred references dealing with the various phases of the problem of teaching arithmetic.

Since arithmetic has been taught primarily in the elementary grades this volume is concerned primarily with elementary instruction. For this reason the readers of the NATIONAL MATHEMATICS MAGAZINE will probabiy be most interested in what Harry E. Benz has to say in the chapter devoted to *Arithmetic in the Senior High School.* He is of the opinion that the recent tendency to introduce arithmetic into the curriculum of the high school is a commendable one. He reviews history indicating that the subject was formerly included in the college curriculum as a separate subject and was not then considered as a study only for children. He also discusses the advantages from the standpoint of mental maturity and from the standpoint of the new interest in and need for arithmetic which arises at the adolescent and adult level. Benz does not think that the introduction of arithmetic in the high school curriculum means necessarily the introduction of special courses but may only mean the incorporation of such materials in other mathematics courses, in courses in other fields (economics, government,

home economics, industrial arts, et. al.), or in activities of a type which transcend the usual subject matter boundaries. This chapter attempts to review the type of arithmetic materials recommended for inclusion in high school, including (1) applications of the four fundamental processes to concrete situations, (2) the meaning of computational processes, (3) applications of number theory to everyday experiences, and (4) materials introduced primarily to stimulate interest in the field of arithmetic.

The report when viewed as a whole is not characterized by an entirely consistent point of view. In most instances the writer of a chapter has tried to mention important differences of opinion and then to give his own conclusions. These conclusions are not those of the entire Committee but of the individuals who wrote each chapter,—Benz, Bond, Brownell, Brueckner, Buckingham, Buswell, Grossnickle, McConnell, Morton, Sauble, Stretch Sueltz, Thiele, Wheat, and Wren. At a time when differences in educational theory and practice are popular this presentation of differences and preservation of individuality is commendable. One has the feeling, however, that a greater measure of group writing might have resulted in a sharpening of issues and a more comprehensive discussion of some of the important problems without the eliminating of desirable differences in opinion.

It must also be said that the different chapters do not seem to be equal in quality and particularly in their help to the reader in evaluating the newer trends in the field and in understanding the place of arithmetic in new patterns of the curriculum. In a few chapters newer trends are ignored while in others the treatment is too brief to be helpful. Original plans for the volume included contributions by two writers who were to stress these newer trends. The omission of materials by these authors is probably responsible in part for the void. We are inclined to the opinion, however, that greater attention to these innovations should have been given in the chapters now included in the volume.

Northwestern University. KENNETH L. HEATON.

The Mathematics of Finance By L. R. Perkins and R. M. Perkins. John Wiley and Sons, Inc., New York, 1941. xx+321 pages. $3.25.

This book is somewhat more comprehensive than many of the texts with similar titles. In a note to prospective users the authors state: "Experience with this text at Middlebury has shown that it is sufficient for a full-year, three-hour-per-week course with enough exercise material to insure that the assignments for two successive years shall contain very little duplication." Selections for a one-semester course are also suggested.

Chapter headings are as follows: *Foundations of Finance: Interest and Discount, Annually Paid Annuities, Annuities Paid in Installments, Paying Debts, Depreciation, Bonded Debts and Bonds as Investments, Building and Loan Associations, Choice; The Foundations of Probability, Probability, One of the Foundations of Life Annuities and Life Insurance, Life Annuities, Life Insurance.* In addition, there is "a complete list of tables, some of them here published for the first time." The list does not include logarithmic tables. There is an index, and answers are given after most of the probelms.

Authors in this field of text books have not succeeded, it seems, in avoiding whole pages of formulas in the treatment of the general annuity. In the text under review, for example, pages 40-41 are likely to be rather dull to the reader who knows mathematics, and quite formidable to the student who is mathematically immature and lacks the knack of cutting through details to get at the essentials. The title of Chapter VIII seems unfortunate. *Foundations of Probability* connotes, to the reviewer at least, some

sort of axiomatic approach to the subject. What the authors mean is that the subject of *probability* is approached along the avenue of *permutations and combinations*.

An attractive feature of the book is the emphasis on the principles of dated values and the use of diagrams as an effective teaching device. The chapter on Bonds is motivated by an introduction with a pleasant flavor of practicality. In fact, throughout the text attention is given to financial practice. The exposition is clear and the press work is very good.

University of Wisconsin at Milwaukee. J. F KENNEY.

Introduction to the Theory of Equations. By Nelson Bush Conkwright. Ginn and Company, Boston, 1941. viii+214 pages. $2.00.

Although the author presupposes some knowledge of the calculus, this book can be taught satisfactorily to students not having such knowledge by making use of Appendix C. Estimated time required to complete the book if it is used as the basis for an advanced under-graduate course is from four and one-half to six months, three times a week.

The chapter headings are: *Complex Numbers; General Theorems on Algebraic Equations; Preliminary Examination of an Equation; Elementary Methods of Solution; Cubic and Quartic Equations; The Theorems of Sturm and Budan; Numerical Approximation to the Roots; Determinants; Simultaneous Linear Equations; Symmetric Functions; Resultants, Discriminants, and Elimination;* and *The Græffe Method.* Also are included Appendices A and G on the *Highest Common Factor, Unique Factorization, the Derivative and Taylor's Series, Cube Roots, Special Cases of Homogeneous Linear Equations, Remarks on Elimination,* and a Supplement to *Horner's Method.* Answers are given to about half of the exercises.

Points worthy of particular mention are: the grading of the exercises, from the easy to the more difficult; general remarks of interest, historical and otherwise, scattered throughout the book; the excellent discussion on numerical approximation to the roots, with a good selection of exercises and problems requiring the solution of transcendental equations; and the inclusion of the Græffe method for finding irrational roots, which, although somewhat tedious, nevertheless seems the best method for finding irrational imaginary roots of polynomial equations.

North Carolina State College. C. G. MUMFORD.

The Second Yearbook of Research and Statistical Methodology. By Oscar K. Buros, (ed). Highland Park N. J., The Gryphon Press, 1941. 383 pages.

When Buros published the *Nineteen Forty Mental Measurements Yearbook* he omitted from it the section on research and statistical books which was included in earlier Yearbooks. This new volume of 383 pages, entitled *Research and Statistical Methodology*, is the substitute for this section in the earlier volume.

In the preparation of the new publication the author selected the 359 books in the field of research and statistical methodology which have been published in English since 1932 (including publications in Australia, Belgium, Canada, China, England, Germany, Holland, India, Norway, Scotland, Sweden, Switzerland, Union of South Africa, and the United States). Books previously listed in Buros' 1938 publication were included if new reviews have appeared since the earlier publication was prepared. Actually 234 books are listed for the first time and 125 titles are repeated. These

volumes are not limited to any one field of research but include specialized publications related to agriculture, anthrolopogy, biology and bacteriology, medicine and related professions, business and economics, education and psychology, political science and public administration, social sciences, and physical sciences, engineering, etc.

After the list of books was prepared a search for reviews of these volumes was made and the critical portions from some 1652 of these reviews excerpted. This volume, therefore, includes the critical portions of all reviews which could be located for the 359 volumes on research methodology. It also includes a directory and index of periodicals and publishers, and indexes of titles, names, and books.

The author's purposes in this ambitious undertaking are stated as follows: "(a) To make students and teachers of statistics more keenly aware of the inadequacy of much of what is now presented in textbooks and classes.... (b) To help students, teachers, and librarians to select textbooks with greater discrimination. (c) To point out to students and teachers the weak and strong points of particular books. (d) To assist more advanced students in keeping abreast of modern developments in monograph and textbook writing and criticism. (e) To encourage research workers to consider and examine methodology books intended for workers in other fields and also books on general history of science, scientific method, and the social relations of science. (f) To emphasize that there are usually marked differences of opinion even among the more advanced students of statistical theory in their appraisal of a particular book. (g) To indicate the vast extension of fields in which statistical techniques are being found useful.... (h) To discourage the writing and publication of stereotyped textbooks.... (i) To make readily available important and provocative statements which, though appearing in book reviews, have considerable value entirely apart from a consideration of the book under review. (j) To improve the quality of reviews by stimulating editors to take greater pains to choose competent reviewers.... (k) To improve the quality of book reviews by stimulating reviewers 'to take their responsibilities more seriously'...."

The effort is a commendable one and the publication will prove itself of great value. Its limitations are primarily the limitations of the reviews upon which it is based. There is every evidence that the quotations have been selected with a desire to give the reader an unbiased judgment of the volumes in question. If a clear evaluation of each of the volumes is not always given—and we feel that such clarity is often lacking—it would seem to be due to the uncritical character of the reviews themselves.

Another defect in the volume could be easily corrected in forthcoming editions. The classified index does not seem to make it easy to locate volumes which have a bearing on a wide range of important topics. If the reader comes to the *Yearbook* to secure advice on a particular volume which he has in mind, then he turns easily to the pertinent section. If on the other hand he comes to gain information regarding the best books dealing with some particular method or problem of research without already knowing the titles of these books, he finds the topical index to be entirely inadequate.

In spite of these two limitations we feel that the volume is an important addition to professional literature. Mr. Buros is deserving of both praise and gratitude for his accomplishment.

Northwestern University. KENNETH L. HEATON.

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Introduction to Algebraic Theories. By A. Adrian Albert. Published by the University of Chicago Press, Chicago, 1941. $1.75.

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Vocational Choices of Students from Cities, Towns, and Farms. By E. Donald Sisson. Studies from the Freshman Division, College of Arts and Sciences, L. S. U., No. 5, Oct. 7, 1941. 4 pages.